CHAPTER 42

EXERGY ANALYSIS AND ENTROPY GENERATION MINIMIZATION

Adrian Bejan
Department of Mechanical Engineering and Materials Science
Duke University
Durham, North Carolina

42.1 INTRODUCTION

In this chapter, we review two important methods that account for much of the newer work in engineering thermodynamics and thermal design and optimization. The method of *exergy analysis* rests on thermodynamics alone. The first law, the second law, and the environment are used simultaneously in order to determine (i) the theoretical operating conditions of the system in the reversible limit and (ii) the entropy generated (or exergy destroyed) by the actual system, that is, the departure from the reversible limit. The focus is on *analysis*. Applied to the system components individually, exergy analysis shows us quantitatively how much each component contributes to the overall irreversibility of the system.[1-3]

Entropy generation minimization (EGM) is a method of *modeling* and *optimization*. The entropy generated by the system is first developed as a function of the physical characteristics of the system (dimensions, materials, shapes, constraints). An important preliminary step is the construction of a system model that incorporates not only the traditional building blocks of engineering thermodynamics (systems, laws, cycles, processes, interactions), but also the fundamental principles of fluid mechanics, heat transfer, mass transfer and other transport phenomena. This combination makes the model "realistic" by accounting for the inherent irreversibility of the actual device. Finally, the minimum entropy generation design ($S_{gen,min}$) is determined for the model, and the approach of any other design (S_{gen}) to the limit of realistic ideality represented by $S_{gen,min}$ is monitored in terms of the entropy generation number $N_S = S_{gen}/S_{gen,min} > 1$.

To calculate S_{gen} and minimize it, the analyst does not need to rely on the concept of exergy. The EGM method represents an important step beyond thermodynamics. It is a new method[4] that combines thermodynamics, heat transfer, and fluid mechanics into a powerful technique for modeling and optimizing real systems and processes. The use of the EGM method has expanded greatly during the last two decades.[5]

SYMBOLS AND UNITS

a specific nonflow availability, J/kg

A nonflow availability, J

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A	area, m^2
b	specific flow availability, J/kg
B	flow availability, J
B	duty parameter for plate and cylinder
B_s	duty parameter for sphere
B_0	duty parameter for tube
Be	dimensionless group, $\dot{S}'''_{\text{gen},\Delta T}/(\dot{S}'''_{\text{gen},\Delta T} + \dot{S}'''_{\text{gen},\Delta P})$
c_p	specific heat at constant pressure, J/(kg · K)
C	specific heat of incompressible substance, J/(kg · K)
C	heat leak thermal conductance, W/K
C^*	time constraint constant, sec/kg
D	diameter, m
e	specific energy, J/kg
E	energy, J
$\bar{e}_{\text{ch}}$	specific flow chemical exergy, J/kmol
$\bar{e}_t$	specific total flow exergy, J/kmol
e_x	specific flow exergy, J/kg
$\bar{e}_x$	specific flow exergy, J/kmol
E_Q	exergy transfer via heat transfer, J
$\dot{E}_W$	exergy transfer rate, W
E_x	flow exergy, J
EGM	the method of entropy generation minimization
f	friction factor
F_D	drag force, N
g	gravitational acceleration, m/sec^2
G	mass velocity, $kg/(sec \cdot m^2)$
h	specific enthalpy, J/kg
h	heat transfer coefficient, $W/(m^2K)$
h°	total specific enthalpy, J/kg
H°	total enthalpy, J
k	thermal conductivity, W/(m K)
L	length, m
m	mass, kg
$\dot{m}$	mass flow rate, kg/sec
M	mass, kg
N	mole number, kmol
$\dot{N}$	molal flow rate, kmol/sec
N_S	entropy generation number, $S_{\text{gen}}/S_{\text{gen,min}}$
Nu	Nusselt number
N_{tu}	number of heat transfer units
P	pressure, N/m^2
Pr	Prandtl number
q'	heat transfer rate per unit length, W/m
Q	heat transfer, J
$\dot{Q}$	heat transfer rate, W
r	dimensionless insulation resistance
R	ratio of thermal conductances
Re_D	Reynolds number
s	specific entropy, J/(kg · K)
S	entropy, J/K
S_{gen}	entropy generation, J/K
$\dot{S}_{\text{gen}}$	entropy generation rate, W/K
$\dot{S}'_{\text{gen}}$	entropy generation rate per unit length, W/(m · K)

$\dot{S}'''_{gen}$	entropy generation rate per unit volume, W/(m³ K)
t	time, sec
t_c	time constraint, sec
T	temperature, K
U	overall heat transfer coefficient, W/(m² K)
U_∞	free stream velocity, m/sec
v	specific volume, m³/kg
V	volume, m³
$\mathbf{V}$	velocity, m/sec
$\dot{W}$	power, W
x	longitudinal coordinate, m
z	elevation, m
ΔP	pressure drop, N/m²
ΔT	temperature difference, K
η	first law efficiency
η_{II}	second law efficiency
θ	dimensionless time
μ	viscosity, kg/(sec · m)
μ_i^*	chemical potentials at the restricted dead state, J/kmol
$\mu_{0,i}$	chemical potentials at the dead state, J/kmol
ν	kinematic viscosity, m²/sec
ξ	specific nonflow exergy, J/kg
Ξ	nonflow exergy, J
Ξ_{ch}	nonflow chemical exergy, J
Ξ_t	nonflow total exergy, J
ρ	density, kg/m³

Subscripts

$(\ )_B$	base
$(\ )_C$	collector
$(\ )_C$	Carnot
$(\ )_H$	high
$(\ )_L$	low
$(\ )_m$	melting
$(\ )_{max}$	maximum
$(\ )_{min}$	minimum
$(\ )_{opt}$	optimal
$(\ )_p$	pump
$(\ )_{rev}$	reversible
$(\ )_t$	turbine
$(\ )_0$	environment
$(\ )_\infty$	free stream

42.2 PHYSICAL EXERGY

Figure 42.1 shows the general features of an open thermodynamic system that can interact thermally ($\dot{Q}_0$) and mechanically ($P_0\, dV/dt$) with the atmospheric temperature and pressure reservoir (T_0, P_0). The system may have any number of inlet and outlet ports, even though only two such ports are illustrated. At a certain point in time, the system may be in communication with any number of additional temperature reservoirs ($T_1, \ldots, T_n$), experiencing the instantaneous heat transfer interactions, $\dot{Q}_1, \ldots, \dot{Q}_n$. The work transfer rate $\dot{W}$ represents all the possible modes of work transfer, specifically, the work done on the atmosphere ($P_0\, dV/dt$) and the remaining (useful, deliverable) portions such as $P\, dV/dt$, shaft work, shear work, electrical work, and magnetic work. The useful part is known as available work (or simply exergy) or, on a unit time basis,

$$\dot{E}_W = \dot{W} - P_0 \frac{dV}{dt}$$

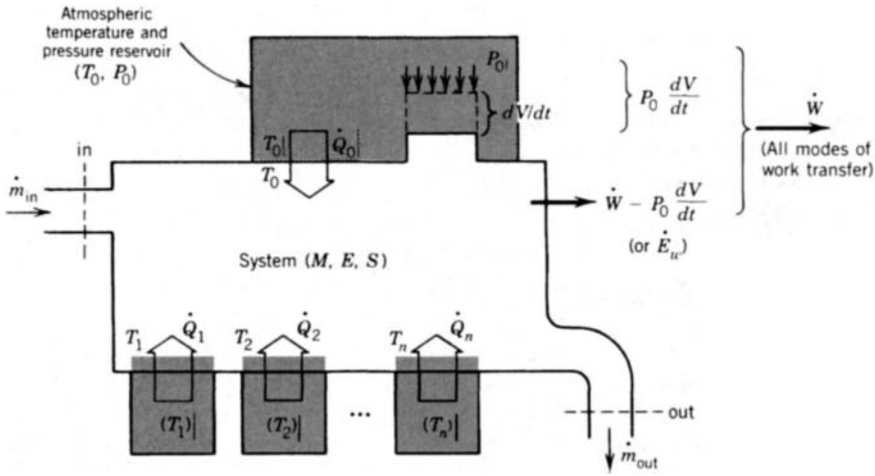

Fig. 42.1 Open system in thermal and mechanical communication with the ambient. (From A. Bejan, *Advanced Engineering Thermodynamics*. © 1997 John Wiley & Sons, Inc. Reprinted by permission.)

The first law and the second law of thermodynamics can be combined to show that the available work transfer rate from the system of Fig. 42.1 is given by the $\dot{E}_W$ equation:[1-3]

$$\dot{E}_W = \underbrace{-\frac{d}{dt}(E - T_0S + P_0V)}_{\substack{\text{Accumulation}\\ \text{of nonflow exergy}}} + \underbrace{\sum_{i=1}^{n}\left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i}_{\substack{\text{Exergy transfer}\\ \text{via heat transfer}}}$$

$$+ \underbrace{\sum_{\text{in}} \dot{m}(h^\circ - T_0 s)}_{\substack{\text{Intake of}\\ \text{flow exergy via}\\ \text{mass flow}}} - \underbrace{\sum_{\text{out}} \dot{m}(h^\circ - T_0 s)}_{\substack{\text{Release of}\\ \text{flow exergy via}\\ \text{mass flow}}} - \underbrace{T_0\dot{S}_{\text{gen}}}_{\substack{\text{Destruction}\\ \text{of exergy}}}$$

where E, V, and S are the instantaneous energy, volume, and entropy of the system, and h° is shorthand for the specific enthalpy plus the kinetic and potential energies of each stream, $h^\circ = h + \frac{1}{2}V^2 + gz$. The first four terms on the right-hand side of the $\dot{E}_W$ equation represent the energy rate delivered as useful power (to an external user) in the limit of reversible operation ($\dot{E}_{W,\text{rev}}$, $\dot{S}_{\text{gen}} = 0$). It is worth noting that the $\dot{E}_W$ equation is a restatement of the Gouy–Stodola theorem (see Section 41.4), or the proportionality between the rate of exergy (work) destruction and the rate of entropy generation

$$\dot{E}_{W,\text{rev}} - \dot{E}_W = T_0\dot{S}_{\text{gen}}$$

A special exergy nomenclature has been devised for the terms formed on the right side of the $\dot{E}_W$ equation. The exergy content associated with a heat transfer interaction (Q_i, T_i) and the environment (T_0) is the *exergy of heat transfer*,

$$E_{Q_i} = Q_i\left(1 - \frac{T_0}{T_i}\right)$$

This means that the heat transfer with the environment (Q_0, T_0) carries zero exergy relative to the environment T_0.

Associated with the system extensive properties (E, S, V) and the two specified intensive properties of the environment (T_0, P_0) is a new extensive property: the thermomechanical or physical *nonflow availability*,

$$A = E - T_0S + P_0V$$
$$a = e - T_0s + P_0v$$

Let A_0 represent the nonflow availability when the system is at the *restricted dead state* (T_0, P_0), that is, in thermal and mechanical equilibrium with the environment, $A_0 = E_0 - T_0S_0 + P_0V_0$. The difference between the nonflow availability of the system in a given state and its nonflow availability in the restricted dead state is the thermomechanical or physical *nonflow exergy*,

$$\Xi = A - A_0 = E - E_0 - T_0(S - S_0) + P_0(V - V_0)$$
$$\xi = a - a_0 = e - e_0 - T_0(s - s_0) + P_0(v - v_0)$$

The nonflow exergy represents the most work that would become available if the system were to reach its restricted dead state reversibly, while communicating thermally only with the environment. In other words, the nonflow exergy represents the exergy content of a given closed system relative to the environment.

Associated with each of the streams entering or exiting an open system is the thermomechanical or physical *flow availability*,

$$B = H^\circ - T_0S$$
$$b = h^\circ - T_0s$$

At the restricted dead state, the nonflow availability of the stream is $B_0 = H_0^\circ - T_0S_0$. The difference $B - B_0$ is known as the thermomechanical or physical *flow exergy* of the stream,

$$E_x = B - B_0 = H^\circ - H_0^\circ - T_0(S - S_0)$$
$$e_x = b - b_0 = h^\circ - h_0^\circ - T_0(s - s_0)$$

Physically, the flow exergy represents the available work content of the stream relative to the restricted dead state (T_0, P_0). This work could be extracted in principle from a system that operates reversibly in thermal communication only with the environment (T_0), while receiving the given stream ($\dot{m}$, h°, s) and discharging the same stream at the environmental pressure and temperature ($\dot{m}$, h_0°, s_0).

In summary, the $\dot{E}_W$ equation can be rewritten more simply as

$$\dot{E}_W = -\frac{d\Xi}{dt} + \sum_{i=1}^{n} \dot{E}_{Q_i} + \sum_{\text{in}} \dot{m}e_x - \sum_{\text{out}} \dot{m}e_x - T_0\dot{S}_{\text{gen}}$$

Examples of how these exergy concepts are used in the course of analyzing component by component the performance of complex systems can be found in Refs. 1–3. Figure 42.2 shows one such example.[1] The upper part of the drawing shows the traditional description of the four components of a simple Rankine cycle. The lower part shows the exergy streams that enter and exit each component, with the important feature that the heater, the turbine and the cooler destroy significant portions (shaded, fading away) of the entering exergy streams. The numerical application of the $\dot{E}_W$ equation to each component tells the analyst the exact widths of the exergy streams to be drawn in Fig. 42.2. In graphical or numerical terms, the "exergy wheel" diagram[1] shows not only *how much* exergy is being destroyed but also *where*. It tells the designer how to rank order the components as candidates for optimization according to the method of *entropy generation minimization* (Sections 42.4–42.9).

To complement the traditional (first law) energy conversion efficiency, $\eta = (\dot{W}_t - \dot{W}_p)/\dot{Q}_H$ in Fig. 42.2, exergy analysis recommends as figure of merit the *second law efficiency*,

$$\eta_{\text{II}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{E}_{QH}}$$

where $\dot{W}_t - \dot{W}_p$ is the net power output (i.e., $\dot{E}_W$ earlier in this section). The second law efficiency can have values between 0 and 1, where 1 corresponds to the reversible limit. Because of this limit, η_{II} describes very well the fundamental difference between the method of exergy analysis and the method of entropy generation minimization (EGM), because in EGM the system always operates *irreversibly*. The question in EGM is how to change the system such that its $\dot{S}_{\text{gen}}$ value (always finite) approaches the minimum $\dot{S}_{\text{gen}}$ allowed by the system constraints.

42.3 CHEMICAL EXERGY

Consider now a nonflow system that can experience heat, work, and mass transfer in communication with the environment. The environment is represented by T_0, P_0, and the n chemical potentials $\mu_{0,i}$

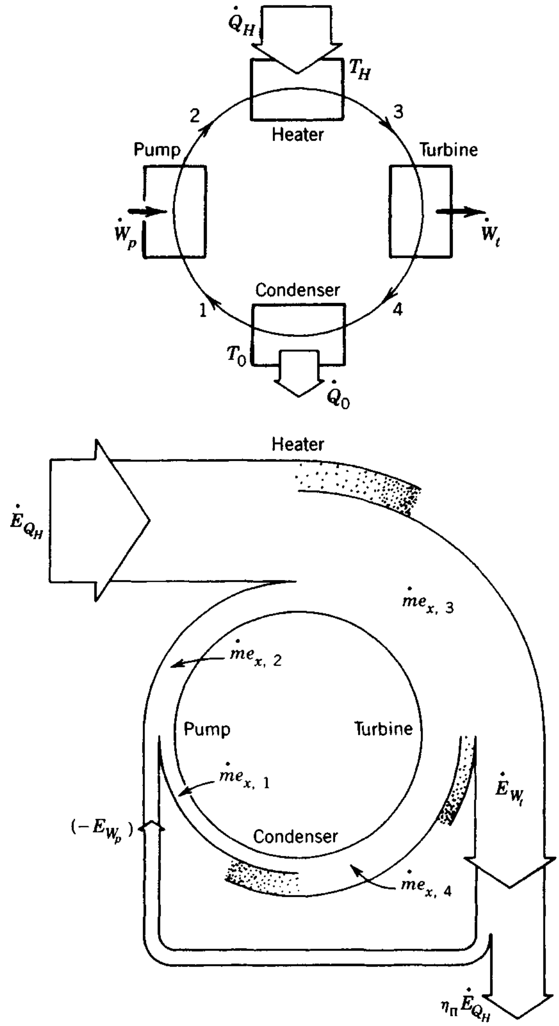

Fig. 42.2 The exergy wheel diagram of a simple Rankine cycle. Top: the traditional notation and energy interactions. Bottom: the exergy flows and the definition of the second law efficiency. (From A. Bejan, *Advanced Engineering Thermodynamics*. © 1997 John Wiley & Sons, Inc. Reprinted by permission.)

of the environmental constituents that are also present in the system. Taken together, the $n + 2$ intensive properties of the environment (T_0, P_0, $\mu_{0,i}$) are known as the *dead state.*

Reading Fig. 42.3 from left to right, we see the system in its initial state represented by E, S, V and its composition (mole numbers $N_1, \ldots, N_n$), and its $n + 2$ intensities (T, P, μ_i). The system can reach its dead state in two steps. In the first, it reaches only thermal and mechanical equilibrium with the environment (T_0, P_0), and delivers the nonflow exergy Ξ defined in the preceding section. At the end of this first step, the chemical potentials of the constituents have changed to μ_i^* ($i = 1, \ldots, n$). During the second step, mass transfer occurs (in addition to heat and work transfer) and, in the end, the system reaches chemical equilibrium with the environment, in addition to thermal and mechanical equilibrium. The work made available during this second step is known as *chemical exergy*,[1-3]

$$\Xi_{ch} = \sum_{i=1}^{n} (\mu_i^* - \mu_{0,i})N_i$$

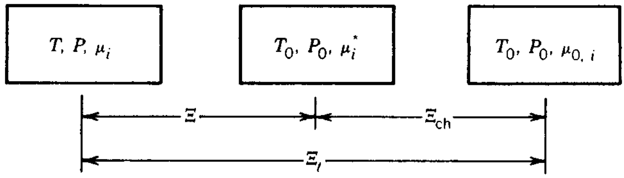

Fig. 42.3 The relationship between the nonflow total (Ξ_t), physical (Ξ), and chemical (Ξ_{ch}) exergies. (From A. Bejan, *Advanced Engineering Thermodynamics*. © 1997 John Wiley & Sons, Inc. Reprinted by permission.)

The total exergy content of the original nonflow system (E, S, V, N_i) relative to the environmental dead state (T_0, P_0, $\mu_{0,i}$) represents the *total nonflow exergy,*

$$\Xi_t = \Xi + \Xi_{ch}$$

Similarly, *the total flow exergy of* a mixture stream of total molal flow rate $\dot{N}$ (composed of n species, with flow rates $\dot{N}_i$) and intensities T, P and μ_i ($i = 1, \ldots, n$) is, on a mole of mixture basis,

$$\bar{e}_t = \bar{e}_x + \bar{e}_{ch}$$

where the physical flow exergy $\bar{e}_x$ was defined in the preceding section, and $\bar{e}_{ch}$ is the *chemical exergy* per mole of mixture,

$$\bar{e}_{ch} = \sum_{i=1}^{n} (\mu_i^* - \mu_{0,i}) \frac{\dot{N}_i}{\dot{N}}$$

In the $\bar{e}_{ch}$ expression μ_i^* ($i = 1, \ldots, n$) are the chemical potentials of the stream constituents at the restricted dead state (T_0, P_0). The chemical exergy is the additional work that could be extracted (reversibly) as the stream evolves from the restricted dead state to the dead state (T_0, P_0, $\mu_{0,i}$) while in thermal, mechanical, and chemical communication with the environment. Applications of the concepts of chemical exergy and total exergy can be found in Refs. 1–3.

42.4 ENTROPY GENERATION MINIMIZATION

The EGM method[4,5] is distinct from exergy analysis, because in exergy analysis the analyst needs only the first law, the second law, and a convention regarding the values of the intensive properties of the environment. The critically new aspects of the EGM method are system modeling, the development of S_{gen} as a function of the physical parameters of the model, and the *minimization* of the calculated entropy generation rate. To minimize the irreversibility of a proposed design, the engineer must use the relations between temperature differences and heat transfer rates, and between pressure differences and mass flow rates. The engineer must relate the degree of thermodynamic nonideality of the design to the physical characteristics of the system, namely, to finite dimensions, shapes, materials, finite speeds, and finite-time intervals of operation. For this, the engineer must rely on heat transfer and fluid mechanics principles, in addition to thermodynamics. Only by varying one or more of the physical characteristics of the system can the engineer bring the design closer to the operation characterized by minimum entropy generation subject to finite-size and finite-time constraints.

The modeling and optimization progress made in EGM is illustrated by some of the simplest and most fundamental results of the method, which are reviewed in the following sections. The structure of the EGM field is summarized in Fig. 42.4 by showing on the vertical the expanding list of applications. On the horizontal, we see the two modeling approaches that are being used. One approach is to focus from the start on the total system, to "divide" the system into compartments that account for one or more of the irreversibility mechanisms, and to declare the "rest" of the system irreversibility-free. In this approach, success depends fully on the modeler's intuition, as there are not one-to-one relationships between the assumed compartments and the pieces of hardware of the real system.

In the alternative approach (from the right in Fig. 42.4), modeling begins with dividing the system into its real components, and recognizing that each component may contain large numbers of one or more elemental features. The approach is to minimize S_{gen} in a fundamental way at each level, starting from the simple and proceeding toward the complex. Important to note is that when a component or elemental feature is imagined separately from the larger system, the quantities assumed specified at the points of separation act as constraints on the optimization of the smaller system. The principle

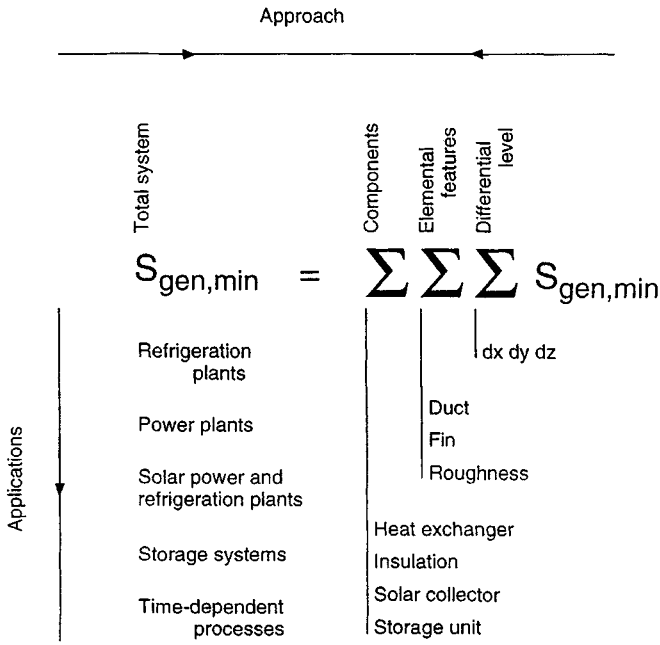

Fig. 42.4 Approaches and applications of the method of entropy generation minimization (EGM). (Reprinted by permission from A. Bejan, *Entropy Generation Minimization*. Copyright CRC Press, Boca Raton, Florida. © 1996.)

of thermodynamic isolation (Ref. 5, p. 125) must be kept in mind during the later stages of the optimization procedure, when the optimized elements and components are integrated into the total system, which itself is optimized for *minimum cost* in the final stage.[3]

42.5 CRYOGENICS

The field of low-temperature refrigeration was the first where EGM became an established method of modeling and optimization. Consider a path for heat leak ($\dot{Q}$) from room temperature (T_H) to the cold end (T_L) of a low-temperature refrigerator or liquefier. Examples of such paths are mechanical supports, insulation layers without or with radiation shields, counterflow heat exchangers, and electrical cables. The total rate of entropy generation associated with the heat leak path is

$$\dot{S}_{gen} = \int_{T_L}^{T_H} \frac{\dot{Q}}{T^2}\, dT$$

where $\dot{Q}$ is in general a function of the local temperature T. The proportionality between the heat leak and the local temperature gradient along its path, $\dot{Q} = kA\,(dT/dx)$, and the finite size of the path [length L, cross section A, material thermal conductivity $k(T)$] are accounted for by the integral constraint

$$\int_{T_L}^{T_H} \frac{k(T)}{\dot{Q}(T)}\, dT = \frac{L}{A} \quad \text{(constant)}$$

The optimal heat leak distribution that minimizes $\dot{S}_{gen}$ subject to the finite-size constraint is[4,5]

$$\dot{Q}_{opt}(T) = \left(\frac{A}{L}\int_{T_L}^{T_H} \frac{k^{1/2}}{T}\, dT\right) k^{1/2} T$$

$$\dot{S}_{gen,min} = \frac{A}{L}\left(\int_{T_L}^{T_H} \frac{k^{1/2}}{T}\, dT\right)^2$$

The technological applications of the variable heat leak optimization principle are numerous and important. In the case of a mechanical support, the optimal design is approximated in practice by placing a stream of cold helium gas in counterflow (and in thermal contact) with the conduction path. The heat leak varies as $d\dot{Q}/dT = \dot{m}c_p$, where $\dot{m}c_p$ is the capacity flow rate of the stream. The practical value of the EGM theory is that it guides the designer to an optimal flow rate for minimum entropy generation. To illustrate, if the support conductivity is temperature-independent, then the optimal flow rate is $\dot{m}_{opt} = (Ak/Lc_p) \ln (T_H/T_L)$. In reality, the conductivity of cryogenic structural materials varies strongly with the temperature, and the single-stream intermediate cooling technique can approach $\dot{S}_{gen,min}$ only approximately.[4,5]

Other applications include the optimal cooling (e.g., optimal flow rate of boil-off helium) for cryogenic current leads, and the optimal temperatures of cryogenic radiation shields. The main counterflow heat exchanger of a low-temperature refrigeration machine is another important path for heat leak in the end-to-end direction ($T_H \rightarrow T_L$). In this case, the optimal variable heat leak principle translates into[4,5]

$$\left(\frac{\Delta T}{T}\right)_{opt} = \frac{\dot{m}c_p}{UA} \ln \frac{T_H}{T_L}$$

where ΔT is the local stream-to-stream temperature difference of the counterflow, $\dot{m}c_p$ is the capacity flow rate through one branch of the counterflow, and UA is the fixed size (total thermal conductance) of the heat exchanger. Other EGM applications in the field of cryogenics are reviewed in Refs. 4 and 5.

42.6 HEAT TRANSFER

The field of heat transfer adopted the techniques developed in cryogenic engineering and applied them to a vast selection of devices for promoting heat transfer. The EGM method was applied to complete components (e.g., heat exchangers) and elemental features (e.g., ducts, fins). For example, consider the flow of a single-phase stream ($\dot{m}$) through a heat exchanger tube of internal diameter D. The heat transfer rate per unit of tube length q' is given. The entropy generation rate per unit of tube length is

$$\dot{S}'_{gen} = \frac{q'^2}{\pi k T^2 \text{Nu}} + \frac{32\dot{m}^3 f}{\pi^2 \rho^2 T D^5}$$

where Nu and f are the Nusselt number and the friction factor, $\text{Nu} = hD/k$ and $f = (-dP/dx)\rho D/(2G^2)$ with $G = \dot{m}/(\pi D^2/4)$. The $\dot{S}'_{gen}$ expression has two terms, in order, the irreversibility contributions made by heat transfer and fluid friction. These terms compete against one another such that there is an optimal tube diameter for minimum entropy generation rate,[4,5]

$$\text{Re}_{D,opt} \cong 2B_0^{0.36}\,\text{Pr}^{-0.07}$$

$$B_0 = \frac{q'\dot{m}\rho}{(kT)^{1/2}\,\mu^{5/2}}$$

where $\text{Re}_D = VD/\nu$ and $V = \dot{m}/(\rho\pi D^2/4)$. This result is valid in the range $2500 < \text{Re}_D < 10^6$ and $\text{Pr} > 0.5$. The corresponding entropy generation number is

$$N_S = \frac{\dot{S}'_{gen}}{\dot{S}'_{gen,min}} = 0.856\left(\frac{\text{Re}_D}{\text{Re}_{D,opt}}\right)^{-0.8} + 0.144\left(\frac{\text{Re}_D}{\text{Re}_{D,opt}}\right)^{4.8}$$

where $\text{Re}_D/\text{Re}_{D,opt} = D_{opt}/D$ because the mass flow rate is fixed. The N_S criterion was used extensively in the literature to monitor the approach of actual designs to the optimal irreversible designs conceived subject to the same constraints.[4,5]

The EGM of elemental features was extended to the optimization of augmentation techniques such as extended surfaces (fins), roughened walls, spiral tubes, twisted tape inserts, and full-size heat exchangers that have such features. For example, the entropy generation rate of a body with heat transfer and drag in an external stream (U_∞, T_∞) is

$$\dot{S}_{gen} = \frac{\dot{Q}_B(T_B - T_\infty)}{T_B\,T_\infty} + \frac{F_D\,U_\infty}{T_\infty}$$

where $\dot{Q}_B$, T_B and F_D are the heat transfer rate, body temperature, and drag force. The relation between $\dot{Q}_B$ and temperature difference ($T_B - T_\infty$) depends on body shape and external fluid and flow, and is provided by the field of convective heat transfer.[6] The relation between F_D, U_∞, geometry and fluid type comes from fluid mechanics.[6] The $\dot{S}_{gen}$ expression has the expected two-term structure, which leads to an optimal body size for minimum entropy generation rate.

The simplest example is the selection of the swept length L of a plate immersed in a parallel stream (Fig. 42.5 inset). The results for $\mathrm{Re}_{L,\mathrm{opt}} = U_\infty L_{\mathrm{opt}}/\nu$ are shown in Fig. 42.5 where B is the constraint (duty parameter)

$$B = \frac{\dot{Q}_B/W}{U_\infty(k\mu T_\infty \mathrm{Pr}^{1/3})^{1/2}}$$

and W is the plate dimension perpendicular to the figure. The same figure shows the corresponding results for the optimal diameter of a cylinder in cross flow, where $\mathrm{Re}_{D,\mathrm{opt}} = U_\infty D_{\mathrm{opt}}/\nu$ and B is given by the same equation as for the plate. The optimal diameter of the sphere is referenced to the sphere duty parameter defined by

$$B_s = \frac{\dot{Q}_B}{\nu(k\mu T_\infty \mathrm{Pr}^{1/3})^{1/2}}$$

The fins built on the surfaces of heat exchanges act as bodies with heat transfer in external flow. The size of a fin of given shape can be optimized by accounting for the internal heat transfer characteristics (longitudinal conduction) of the fin, in addition to the two terms (convective heat and fluid flow) shown in the last $\dot{S}_{\mathrm{gen}}$ formula. The EGM method has also been applied to complete heat exchangers and heat exchanger networks. This vast literature is reviewed in Ref. 5. One technological benefit of EGM is that it shows how to select certain dimensions of a device such that the device destroys minimum power while performing its assigned heat and fluid flow duty.

Several computational heat and fluid flow studies recommended that future commercial CFD packages have the capability of displaying entropy generation rate fields (maps) for both laminar and turbulent flows. For example, Paoletti et al.[7] recommend the plotting of contour lines for constant values of the dimensionless group $\mathrm{Be} = \dot{S}'''_{\mathrm{gen},\Delta T}/(\dot{S}'''_{\mathrm{gen},\Delta T} + \dot{S}'''_{\mathrm{gen},\Delta P})$ where $\dot{S}'''_{\mathrm{gen}}$ means local (volumetric) entropy generation rate, and ΔT and ΔP refer to the heat transfer and fluid flow irreversibilities, respectively.

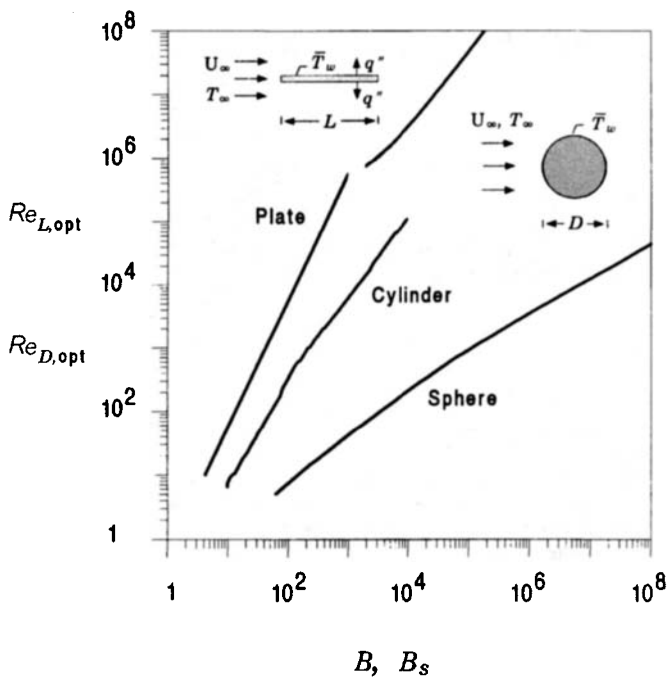

Fig. 42.5 The optimal size of a plate, cylinder and sphere for minimum entropy generation. (From A. Bejan, G. Tsatsaronis, and M. Moran, *Thermal Design and Optimization*. © 1996 John Wiley & Sons, Inc. Reprinted by permission.)

42.7 STORAGE SYSTEMS

In the optimization of time-dependent heating or cooling processes the search is for optimal histories, that is, optimal ways of executing the processes. Consider as a first example the sensible heating of an amount of incompressible substance (mass M, specific heat C), by circulating through it a stream of hot ideal gas ($\dot{m}$, c_p, T_∞) (Fig. 42.6). Initially, the storage material is at the ambient temperature T_0. The total thermal conductance of the heat exchanger placed between the storage material and the gas stream is UA and the pressure drop is negligible. After flowing through the heat exchanger, the gas stream is discharged into the atmosphere. The entropy generated from $t = 0$ until a time t reaches a minimum when t is of the order of $MC/(\dot{m}c_p)$. Charts for calculating the optimal heating (storage) time interval are available in Refs. 4 and 5. For example, when $(T_\infty - T_0) << T_0$, the optimal heating time is given by

$$\theta_{\text{opt}} = \frac{1.256}{1 - \exp(-N_{tu})}$$

where $\theta_{\text{opt}} = t_{\text{opt}}\, \dot{m}c_p/(\text{MC})$ and $N_{\text{tu}} = UA/(\dot{m}c_p)$.

Another example is the optimization of a sensible-heat cooling process subject to an overall time constraint. Consider the cooling of an amount of incompressible substance (M, C) from a given initial temperature to a given final temperature, during a prescribed time interval t_c. The coolant is a stream of cold ideal gas with flow rate $\dot{m}$ and specific heat $c_p(T)$. The thermal conductance of the heat exchanger is UA; however, the overall heat transfer coefficient generally depends on the instantaneous temperature, $U(T)$. The cooling process requires a minimum amount of coolant m (or minimum refrigerator work for producing the cryogen m),

$$m = \int_0^{t_c} \dot{m}(t)\, dt$$

when the gas flow rate has the optimal history[4,5]

$$\dot{m}_{\text{opt}}(t) = \left[\frac{U(T)A}{C^* c_p(T)}\right]^{1/2}$$

In this expression, $T(t)$ is the corresponding optimal temperature history of the object that is being cooled and C^* is a constant that can be evaluated based on the time constraint, as shown in Refs. 4 and 5. The optimal flow rate history result ($\dot{m}_{opt}$) tells the operator that at temperatures where U is small the flow rate should be decreased. Furthermore, since during cooldown the gas c_p increases, the flow rate should decrease as the end of the process nears.

In the case of energy storage by melting there is an optimal melting temperature (i.e., optimal type of storage material) for minimum entropy generation during storage. If T_∞ and T_0 are the temperatures of the heat source and the ambient, the optimal melting temperature of the storage material has the value $T_{m,\text{opt}} = (T_\infty T_0)^{1/2}$.

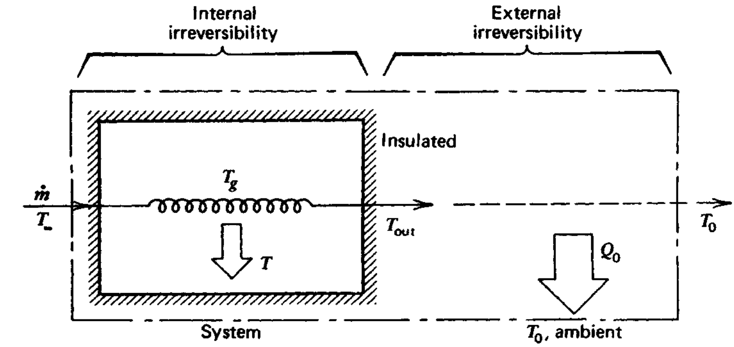

Fig. 42.6 Entropy generation during sensible-heat storage.[4]

42.8 SOLAR ENERGY CONVERSION

The generation of power and refrigeration based on energy from the sun has been the subject of some of the oldest EGM studies, which cover a vast territory. A characteristic of these EGM models is that they account for the irreversibility due to heat transfer in the two temperature gaps (sun-collector and collector-ambient) and that they reveal an optimal *coupling* between the collector and the rest of the plant.

Consider, for example, the steady operation of a power plant driven by a solar collector with convective heat leak to the ambient, $\dot{Q}_0 = (UA)_c(T_c - T_0)$, where $(UA)_c$ is the collector-ambient thermal conductance and T_c is the collector temperature (Fig. 42.7). Similarly, there is a finite size heat exchanger $(UA)_i$ between the collector and the hot end of the power cycle (T), such that the heat input provided by the collector is $\dot{Q} = (UA)_i(T_c - T)$. The power cycle is assumed reversible. The power output $\dot{W} = \dot{Q}\,(1 - T_0/T)$ is maximum, or the total entropy generation rate is minimum, when the collector has the optimal temperature[4,5]

$$\frac{T_{c,\mathrm{opt}}}{T_0} = \frac{\theta_{\max}^{1/2} + R\theta_{\max}}{1 + R}$$

where $R = (UA)_c/(UA)_i$, $\theta_{\max} = T_{c,\max}/T_0$ and $T_{c,\max}$ is the maximum (stagnation) temperature of the collector.

Another type of optimum is discovered when the overall size of the installation is fixed. For example, in an extraterrestrial power plant with collector area A_H and radiator area A_L, if the total area is constrained[1]

$$A_H + A_L = A \quad \text{(constant)}$$

the optimal way to allocate the area is $A_{H,\mathrm{opt}} = 0.35A$ and $A_{L,\mathrm{opt}} = 0.65A$. Other examples of optimal allocation of hardware between various components subject to overall size constraints are given in Ref. 5. The progress on the thermodynamic optimization of solar energy (thermal and photovoltaic) is reviewed in Refs. 1 and 5.

42.9 POWER PLANTS

There are several EGM models and optima of power plants that have fundamental implications. The loss of heat from the hot end of a power plant can be modeled by using a thermal resistance in parallel with an irreversibility-free compartment that accounts for the power output $\dot{W}$ of the actual power plant (Fig. 42.8). The hot-end temperature of the working fluid cycle T_H can vary. The heat input $\dot{Q}_H$ is fixed. The bypass heat leak is proportional to the temperature difference, $\dot{Q}_C = C(T_H - T_L)$, where C is the thermal conductance of the power plant insulation. The power output is maximum (and $\dot{S}_{\mathrm{gen}}$ is minimum) when the hot-end temperature reaches the optimal level[4]

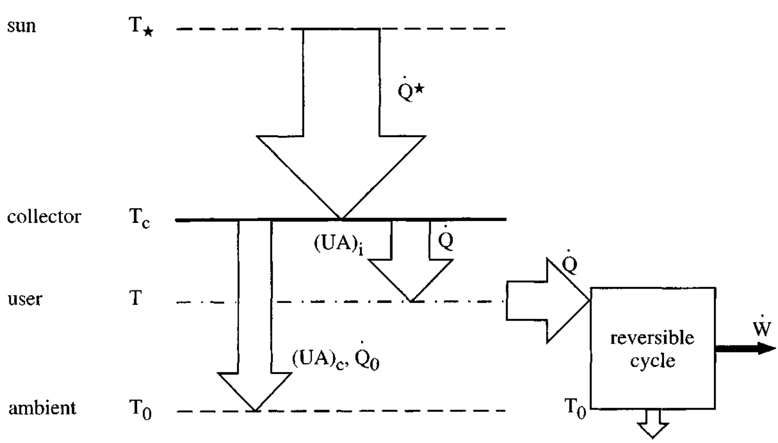

Fig. 42.7 Solar power plant model with collector-ambient heat loss and collector-engine heat exchanger.[4]

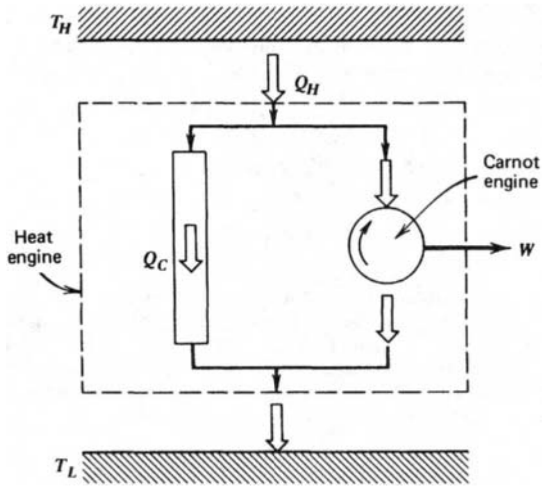

Fig. 42.8 Power plant model with bypass heat leak.[4]

$$T_{H,\text{opt}} = T_L \left(1 + \frac{\dot{Q}_H}{CT_L}\right)^{1/2}$$

The corresponding efficiency ($\dot{W}_{\max}/\dot{Q}_H$) is

$$\eta = \frac{(1 + r)^{1/2} - 1}{(1 + r)^{1/2} + 1}$$

where $r = \dot{Q}_H/(CT_L)$ is a dimensionless way of expressing the size (thermal resistance) of the power plant. An optimal T_H value exists because when $T_H < T_{H,\text{opt}}$, the Carnot efficiency of the power producing compartment is too low, while when $T_H > T_{H,\text{opt}}$, too much of the unit heat input $\dot{Q}_H$ bypasses the power compartment.

Another optimal hot-end temperature is revealed by the power plant model shown in Fig. 42.9 (e.g., Ref. 1, p. 357). The power plant is driven by a stream of hot single-phase fluid of inlet temperature T_H and constant specific heat c_p. The model has two compartments. The one sandwiched

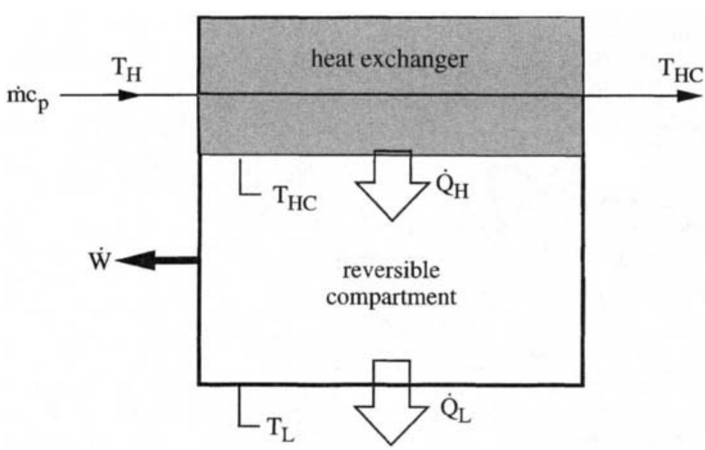

Fig. 42.9 Power plant driven by a stream of hot single-phase fluid.[1,5]

between the heat exchanger surface (T_{HC}) and the ambient (T_L) operates reversibly. The other is a heat exchanger: for simplicity, the area of the T_{HC} surface is assumed sufficiently large that the stream outlet temperature is equal to T_{HC}. The stream is discharged into the ambient. The optimal hot-end temperature for maximum $\dot{W}$ (or minimum $\dot{S}_{gen}$) is[1,5]

$$T_{HC,opt} = (T_H T_L)^{1/2}$$

The corresponding first-law efficiency, $\eta = \dot{W}_{max}/\dot{Q}_H$, is[5]

$$\eta = 1 - \left(\frac{T_L}{T_H}\right)^{1/2}$$

The optimal allocation of a finite heat exchanger inventory between the hot end and the cold end of a power plant is illustrated by the model with two heat exchangers[4] proposed in Fig. 42.10. The heat transfer rates are proportional to the respective temperature differences, $\dot{Q}_H = (UA)_H \Delta T_H$ and $\dot{Q}_L = (UA)_L \Delta T_L$, where the thermal conductances $(UA)_H$ and $(UA)_L$ account for the sizes of the heat exchangers. The heat input $\dot{Q}_L$ is fixed (e.g., the optimization is carried out for one unit of fuel burnt). The role of overall heat exchanger inventory constraint is played by[1]

$$(UA)_H + (UA)_L = UA \quad \text{(constant)}$$

where UA is the total thermal conductance available. The power output is maximized, and the entropy generation rate is minimized, when UA is allocated according to the rule[1,5]

$$(UA)_{H,opt} = (UA)_{L,opt} = \tfrac{1}{2}UA$$

The corresponding maximum efficiency is, as expected, lower than the Carnot efficiency,

$$\eta = 1 - \frac{T_L}{T_H}\left(1 - \frac{4\dot{Q}_H}{T_H UA}\right)^{-1}$$

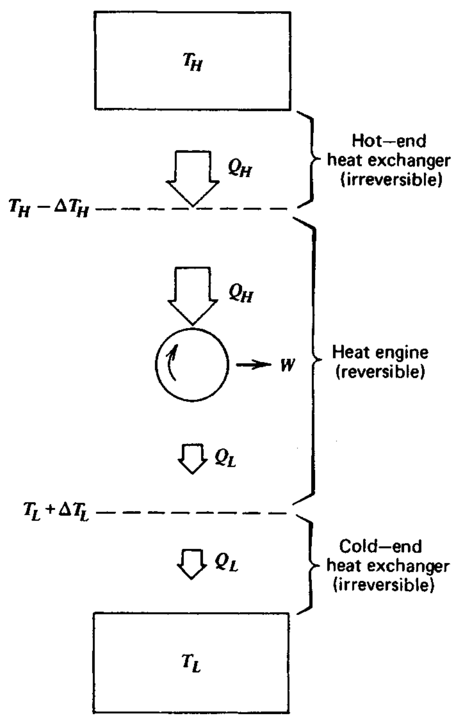

Fig. 42.10 Power plant with two finite-size heat exchangers.[4]

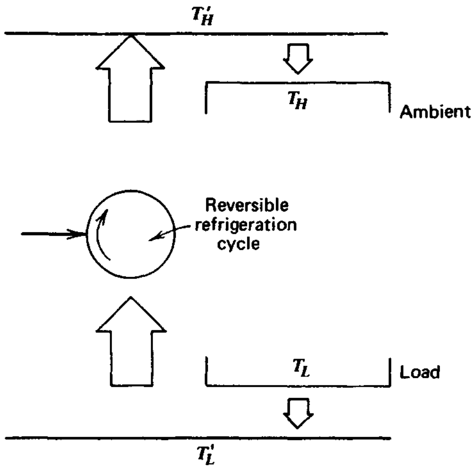

Fig. 42.11 Refrigerator model with two finite-size heat exchangers.[4]

The EGM modeling and optimization progress on power plants is extensive, and is reviewed in Ref. 5. Similar models have also been used in the field of refrigeration, as we saw already in Section 42.5. For example, in a steady-state refrigeration plant with two heat exchangers (Fig. 42.11) subjected to the total *UA* constraint listed above, the refrigerator power input is minimum when *UA* is divided equally among the two heat exchangers, $(UA)_{H,\text{opt}} = \frac{1}{2}UA = (UA)_{L,\text{opt}}$.

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